



Class 1: Algebra and Equations

Warm-up Problem 1

Joshua is reading a 396-page book. He has read the first third of the book only. How many pages does he have left to read to finish the rest of the book? (Source: Gauss 7)

One third of the book is $396 \div 3 = 132$

Joshua has read already read 132 pages.

Pages left = $396 - 132$

= 264

\therefore Joshua has 264 pages left to read.

Warm-up Problem 2

At a factory, 11410 kg of rice is distributed equally into 3260 bags. A family uses 0.25 kg of rice each day. How many days would it take this family to use up one bag of rice? (Source: Gauss 7)

Each bag contains $11410 \div 3260 = 3.5$ kg of rice

Since a family uses 0.25 kg of rice each day, it would take $3.5 \div 0.25 = 14$ days to use up one bag.

\therefore It will take 14 days.

Problem

Karl has 5 chickens and rabbits in total (chickens have 2 legs, rabbits have 4 legs). There are 16 legs in total. How many rabbits are there?

If all 5 animals were chickens, there would be $5 \times 2 = 10$ legs

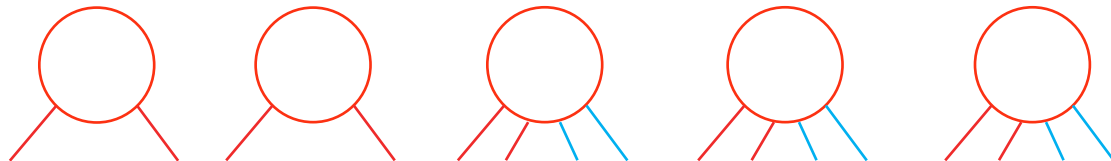
There are $16 - 10 = 6$ missing legs.

Why are there 6 missing legs?

There are 6 missing legs because we assumed that **all 5 animals were chickens**. However, each rabbit has **2 more legs** than a chicken.

For each rabbit, 2 legs have been removed.

There are 6 missing legs in total, and each rabbit needs 2 extra legs, so there are $6 \div 2 = 3$ rabbits.



Problem

Karl has 30 birds. Some of his birds are emus and the rest are chickens. Karl hands out 100 treats to his birds. Each emu gets 2 treats and each chicken gets 4 treats. How many chickens does Karl have? (Source: Gauss 7)

If all 30 birds are emus, he would have to hand out $2 \times 30 = 60$ treats.


Since he hands out exactly 100 treats, there are $100 - 60 = 40$ extra treats.

There is an extra 40 because some treats should belong to the chickens instead.

Each chicken gets 2 more treats than an emu (since $4 - 2 = 2$), so $40 \div 2 = 20$ chickens.

Distributive Property

The distributive property says that


$$a \times (b + c) = ab + ac$$

For example,

$$3 \times (4 + 5) = 3 \times 9 = 27$$

<- Without using Distributive Property

$$3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 12 + 15 = 27$$

<- Using Distributive Property

More Examples of Distributive Property

$$\begin{aligned} 3 \times (y + 3) &= 3y + 3 \times 3 \\ &= 3y + 9 \end{aligned}$$

$$\begin{aligned} 5 \times (k - 1) &= 5k - 5 \times 1 \\ &= 5k - 5 \end{aligned}$$

$$\begin{aligned} 2 \times (2r + 7) &= 2 \times 2r + 2 \times 7 \\ &= 4r + 14 \end{aligned}$$

Exercise

Simplify the following using the Distributive Property:

(a) $3(k + 1)$

(b) $5(y - 4)$

(c) $-6(s + 2)$

(d) $-2(r - 5)$

(a) $3(k + 1) = 3k + 3$

(b) $5(y - 4) = 5y - 20$

(c) $-6(s + 2) = -6s - 12$

(d) $-2(r - 5) = -2r + 10$

Problem

The sum of four numbers is S . Suppose that each of the four numbers is now increased by 1. These four new numbers are added together and then the sum is tripled. What is the value of this final result?
(Source: Variation from Gauss 7)

It doesn't matter what the four numbers are.

If each of the four numbers is increased by 1, the new sum is $S+4$.

Triple means to multiply by 3.

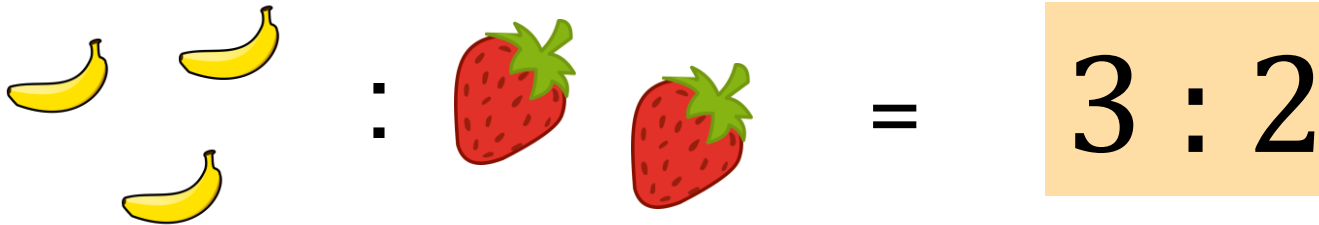
$$\begin{aligned} 3 \times (S + 4) &= 3S + 3 \times 4 && \text{<- Using Distributive Property} \\ &= 3S + 12 \end{aligned}$$

Ratios

What is a Ratio?

A ratio compares two different quantities.


For example, if there are 3 bananas and 2 strawberries, the ratio between bananas and strawberries is 3 : 2.



Simplifying Ratios

Simplifying Ratios

If the two quantities have a **greatest common divisor** of greater than 1, it can be simplified.



The diagram illustrates the simplification of a ratio using fruit icons and numbers. On the left, there are four yellow bananas and two red strawberries, separated by a colon (:). This is followed by an equals sign (=) and the numerical ratio 4 : 2. Above the 4 and 2, there are two curved arrows, one orange and one red, both labeled with the division symbol and the number 2 (÷ 2). This indicates that both numbers are divided by their greatest common divisor, 2. The final result is shown in a light pink box as the simplified ratio 2 : 1.

$$\text{4 Bananas} : \text{2 Strawberries} = 4 : 2 = 2 : 1$$

Exercise

Simplify the following ratios into simplest form:

(a) $6 : 4$

(b) $12 : 9$

Rewrite the following ratios so that there are no decimals and it is in simplest form:

(c) $2.5 : 4$

(d) $1.25 : 3$

(a) 6 and 4 are both divisible by 2, so $6 : 4$ is equivalent to $3 : 2$

(b) 12 and 9 are both divisible by 3, so $12 : 9$ is equivalent to $4 : 3$

(c) Multiply both sides by 2 to get rid of decimals. $2.5 : 4$ is equivalent to $5 : 8$

(d) Multiply both sides by 4 to get rid of decimals. $1.25 : 3$ is equivalent to $5 : 12$

Problem

A parking lot has 25% more cars than trucks. What is the ratio of cars to trucks? (Source: Variation from Gauss 7)

$$25\% = 0.25$$

If there are t trucks, the number of cars is $1.25t$

The ratio of cars : trucks is $1.25t : t$

We can divide by t on both sides of the ratio, so the ratio becomes $1.25 : 1$

Ratios should have integers (no decimals), so we multiply by 4 to get rid of the decimal.

$$1.25 : 1 = 5 : 4$$

Rates

What is a Rate?

A rate is a ratio that compares two quantities with different units of measure.

For example:

A car drove 100 kilometers in 2 hours – the rate is $100\text{km}/2\text{ hours}$ or $50\text{km}/\text{hour}$

Problem

Alex pays \$2.25 to take the bus. Sam pays \$3.00 to take the bus. If they each take the bus 20 times, how much less would Alex pay than Sam in total? (Source: Variation from Gauss 7)

$$3.00 - 2.25 = 0.75$$

Alex pays \$0.75 less than Sam for each bus ride.

$$\text{After 20 times: } 0.75 \times 20 = 15$$

\therefore Alex pays \$15 less than Sam in total.

Problem

Carrie is travelling at a constant speed of 85 km/h. If Carrie is halfway through a 510 km trip, how much longer will the trip take? (Source: Variation from Gauss 7)

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\begin{aligned}\text{Total time needed} &= 510 \div 85 \\ &= 6 \text{ hours}\end{aligned}$$

Since her speed is constant throughout the whole trip, she needs $6 \div 2 = 3$ more hours

\therefore The trip will take 3 more hours.

Averages

The **average** (or **mean**) of some numbers can be found by **adding all the numbers up**, then **dividing by the number of numbers**.

$$\text{Mean} = \text{Sum} \div \text{Count}$$

Example:

Find the mean of 4, 6, and 8.

$$\text{Mean} = \underbrace{(4 + 6 + 8)}_{\text{sum}} \div \underbrace{(3)}_{\text{count}}$$

$$= 18 \div 3$$

$$= 6$$

Problem

The average of 56, 72, and k is 55. Find the value of k. (Source: Variation of Gauss 7)

$$\text{Total} = 55 \times 3 = 165$$

$$56 + 72 + k = 165$$

$$k + 128 = 165$$

$$k = 165 - 128$$

$$k = 37$$